

1.1, 1.2 Notes

1.1: Mathematics and Problem Solving

Four-Step Problem-Solving Process:

1. Understand the problem.
 - a. What is the problem giving me?
 - b. What is the problem asking me?
2. Devise a plan. Some strategies include:
 - a. Look for a pattern.
 - b. Make a table.
 - c. Draw a picture.
3. Carry out the plan.
 - a. Follow your plan.
 - b. Check each step as you go.
4. Look back.
 - a. Check your answer.
 - b. Did you answer the original question?
 - c. Was there another method?
 - d. Are there related problems for which you could use the same techniques?

Answer the following questions.

4. Predict the sum of the page numbers if we use 4 slips of paper.

5. Without using a calculator, determine the value of $1 + 2 + \dots + 24$.

6. Write a formula for the sum of the page numbers with n sheets.

Step 4 - Look Back:

We can check our answers for the 2 case: $1+2+3+4+5+6+7+8=36$.
Did we answer the question?

Let's try a different approach.

Example: What is the sum of the first n positive integers?

Activity: Work with a partner to form a "book" in the following manner:

1. Get 2 slips of paper from the table.
2. Fold each piece of paper in half hamburger style, then put them together to look like a book.
3. Close your book up, and number the outside page 1, flip and number page 2, page 3, etc.

When you are finished, answer the following questions.

1. What is the total of the page numbers on one side of a sheet?

2. What is the total of the page numbers on one whole sheet?

3. What is the total of all the page numbers?

Repeat this for 3 slips of paper.

Let's look at the actual problem solving approach:

Example: If you create a book out of n sheets, what is the sum of the page numbers?

Step 1 - Understand the Problem: What do we need to understand?

Step 2 - Devise a Plan: Our strategy was _____.
How did we do it?

Step 3 - Carry Out the Plan: Using the 2 sheet and 3 sheet example, we figured out how to determine the last page number and how to find (1) the total page numbers on each side of a sheet, (2) the total page numbers on each sheet, and (3) the sum of all page numbers. We then used this information to find the formula for the sum of the pages of a book made from n sheets.

Strategy: Examine a Related Problem

Example: What is the sum of the even numbers less than or equal to 40? Have we done a similar problem before?

1.1, 1.2 Notes

Strategy: Examine a Simpler Case

Example: If there are 10 people in the room and each person shakes every other person's hand, how many handshakes were performed? Nobody shakes hands twice, and shaking your own hand would make you look weird...

Strategy: Make a Table

Example: A wealthy family hired a maid and a gardener. The maid comes in every 2 days, and the gardener comes in every 3 days. If they started on the same day, how many days will go by before they come in on the same day again?

Strategy: Guess and Check

Example: Find two numbers whose product is 42 and whose sum is 17.

Strategy: Identify a sub-goal. In some problems, you may want to find a piece of information that will help you solve the problem first.

Strategy: Make a diagram.

Example: To get some cardio exercise, you climb the stairs in a tall building. You start from the first floor (Floor 1). You then go up 3 floors, down 2 floors, up 7 floors, down 5 floors, and then up 7 floors to stop at the top floor. How many floors does the building have?

Strategy: Work Backward

Example: You have an 80 average on 6 quizzes. Your teacher tells you that you can drop your lowest quiz grade of 30. What is your new average?

Strategy: Use Direct Reasoning.

Example: If two people won 3 games of checkers each, what is the minimum number of games played?

Strategy: Write an Equation. This was discussed heavily in Math 111.

Strategy: Use Indirect Reasoning. Sometimes it is easier to show what the opposite can't happen. An important form of this is the process of elimination.

Example: Andrew, Michael, and Travis played a strategy game. Michael did not come in first place, as usual. Travis beat Michael but he did not come in first place. Who took first, second, and third place?

1.2: Patterns

What appears to be a pattern may need further checking. You need enough data to identify a pattern.

Example: Fill in the following pattern: 13, 19, 25, ____, ____, ____

Example: Fill in 2, 4, ____, ____, ____ in as many ways as you can think of.